

2.3 Properties of Logarithms

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1. Product Property — multiply
2. Quotient Property — divide
3. Power Property — exponents

Product Property:

$$\log_b mn = \log_b m + \log_b n$$

Quotient Property:

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

Power Property:

$$\log_b m^n = n \log_b m$$

$$\log_3 9 + \log_3 24$$

Product

$$m=9$$

$$n=24$$

$$\log_3 (9)(24)$$

$$\log_4 16^3$$

power


$$3 \log_4 16$$

$$\log_b mn = \log_b m + \log_b n$$

(product)

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

(quotient)

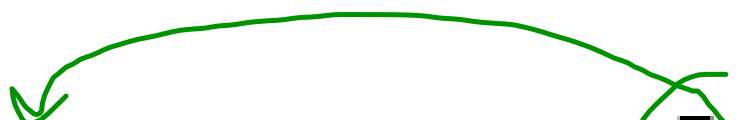

$$\log_b m^n = n \log_b m$$

(power)

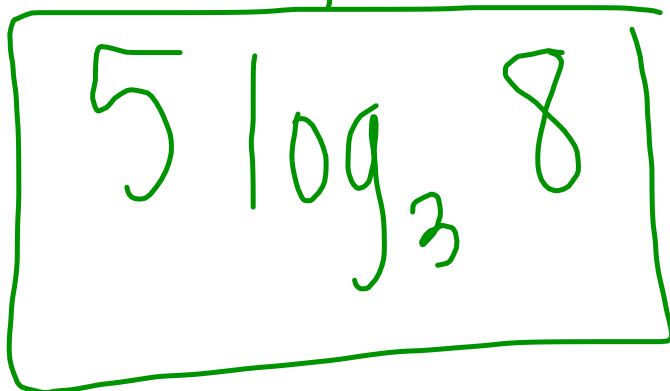
$$\log_2 7 - \log_2 9$$

Quotient

$$\log_2 \frac{7}{9}$$


$$\log_3 8^5$$

power


$$5 \log_3 8$$

$$\log_4 x \ominus \log_4 y$$

Quotient

$$\log_4 \frac{x}{y}$$

$$\log 5 + \log 7$$

$$\frac{1}{2} \log r + \frac{1}{3} \log s - \frac{1}{4} \log t$$

product
Quotient
power

> from left to right

$$\log r^{\frac{1}{2}} + \log s^{\frac{1}{3}} - \log t^{\frac{1}{4}}$$

$$\log r^{\frac{1}{2}} s^{\frac{1}{3}} - \log t^{\frac{1}{4}}$$

$$\log \frac{r^{\frac{1}{2}} s^{\frac{1}{3}}}{t^{\frac{1}{4}}}$$

$$\frac{1}{3} \log 3x + \frac{2}{3} \log 3x$$

product ✓
power ✓

$$\log 3x^{\frac{1}{3}} + \log 3x^{\frac{2}{3}}$$

$$\log(3x^{\frac{1}{3}} \times 3x^{\frac{2}{3}})$$

$$\log_5 y - 4(\log_5 r + 2 \log_5 t)$$

Expand each logarithm. Simplify if possible.

$$\log_3 \underbrace{27}_m \underbrace{x}_n$$

Product
 $\log m + \log n$

$$\log_3(27) + \log_3(x)$$

$$\log \frac{3}{7}$$

Quotient
 $\log m - \log n$

$$\log(3) - \log(7)$$

$$\log_4 \underbrace{y^2} \underbrace{z^3}$$

Product
power - last

$$\log_4 y \quad \text{②} \quad + \quad \log_4 z \quad \text{③}$$

$$2 \log_4 y + 3 \log_4 z$$

$$\log_5 \frac{3^2}{x}$$

power - last
quotient - 1st
(subtract)

$$\log_5 (3^2) - \log_5 (x)$$

$$2 \log_5 3 - \log_5 x$$

$$\log_3 \underbrace{15xy}_{\text{product}}$$

$$\log_3 15 + \log_3 x + \log_3 y$$

