1.5 Remainder Theorem and Roots of an Equation

Day 6 03/31/15

Mar 31-6:27 AM

The **Remainder Theorem** provides a quick way to find the remainder of a polynomial long-division problem.

Here's Why It Works When you divide polynomial $P(x)$ by $D(x)$, you find $P(x) = D(x)Q(x) + R(x)$.

$P(x) = (x - a)Q(x) + R(x)$	Substitute $(x - a)$ for $D(x)$.
$P(a) = (a - a)Q(a) + R(a)$	Evaluate $P(a)$. Substitute a for x.
$= R(a)$	Simplify.

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DONT CHANGE SIGN!!

Given that $P(x) = x^5 - 2x^3 - x^2 + 2$, what is $P(3)$?

3	1	0	−2	−1	0	2
	↓	3	+9	+21	60	180
	1	3	7	20	60	$\boxed{182}$

Remainder

$\boxed{P(3) = 182}$

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Given that $P(x) = 1x^5 - 3x^4 - 28x^3 + 5x + 20$, what is $P(-4)$?

−4	1	−3	−28	0	5	20
		−4	+28	0	0	−20
	1	−7	0	0	5	$\boxed{0}$

$\boxed{P(-4)=0}$

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#38 & #40 → Today 3/31

#1–20 all ← yesterday 3/30

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Rational Root Theorem:

Use $\frac{p}{q}$ to test as points in the equation.

Use synthetic division to test the values. If the remainder is zero, the value is a rational root.

Roots: Points where graph crosses the x-axis

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$\frac{p}{q} = \frac{\text{factors of the constant}}{\text{factors of the leading coefficient}}$

ex: $3x^3 + 10x^2 - 5x + 1$

leading coefficient "q" — constant "p"

factors can be positive or negative

Factors: numbers that multiply to a number

ex: 10 ±1, ±2, ±5, ±10

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List all possible roots:

$x^3 - 4x + 1 = 0$

P: 1 ±1

Q: 1 ±1

$\frac{P}{Q}: \frac{\pm 1}{1} = \pm 1$

$7x^3 - x^2 + 4x + 10 = 0$

P: 10 ±1, ±2, ±5, ±10

Q: 7 ±1, ±7

$\frac{P}{Q}: \frac{\pm 1}{\pm 1}, \frac{\pm 1}{\pm 7}, \frac{\pm 2}{\pm 1}, \frac{\pm 2}{\pm 7}, \frac{\pm 5}{\pm 1}, \frac{\pm 5}{7}, \frac{\pm 10}{1}, \frac{\pm 10}{7}$

$\boxed{\frac{P}{Q}: \pm 1, \pm\frac{1}{7}, \pm 2, \pm\frac{2}{7}, \pm 5, \pm\frac{5}{7}, \pm 10, \pm\frac{10}{7}}$

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List all possible roots:

$3x^3 + 9x - 6 = 0$

p: $\pm 1, \pm 2, \pm 3, \pm 6$

Q: $\pm 1, \pm 3$

$\frac{P}{Q}$: $\pm\frac{1}{1}, \pm\frac{1}{3}, \pm 2, \pm\frac{2}{3}, \pm\frac{3}{1}, \pm\frac{3}{3}, \pm\frac{6}{1}, \pm\frac{6}{3}$

$\boxed{\pm 1, \pm\frac{1}{3}, \pm 2, \pm\frac{2}{3}, \pm 3, \pm 6}$

$4x^3 + 2x - 12 = 0$

p: 12 $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Q: 4 $\pm 1, \pm 2, \pm 4$

$\frac{P}{Q}$: $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{2}{4}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{4}{1}, \pm\frac{4}{2}, \pm\frac{4}{4}, \pm\frac{6}{1}, \pm\frac{6}{2}, \pm\frac{6}{4}, \pm\frac{12}{1}, \pm\frac{12}{2}, \pm\frac{12}{4}$

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$\frac{P}{Q}$: $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 2, \pm 3, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm 4, \pm 6, \pm 12$

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What are the rational roots of $2x^3 - x^2 + 2x + 5 = 0$?

p: 5 $\pm 1, \pm 5$

Q: 2 $\pm 1, \pm 2$

$\frac{P}{Q}$: $\pm 1, \pm\frac{1}{2}, \pm 5, \pm\frac{5}{2}$

Test Points:

1: $2(1)^3 - (1)^2 + 2(1) + 5$

$2 - 1 + 2 + 5$

$1 + 2 + 5 = 8 \neq 0$

-1: $2(-1)^3 - (-1)^2 + 2(-1) + 5$

$-2 - 1 - 2 + 5 = 0$

possible root

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Use synthetic division

$$\begin{array}{r|rrrr} -1 & 2 & -1 & 2 & 5 \\ & \downarrow & -2 & 3 & -5 \\ \hline & 2 & -3 & 5 & 0 \end{array}$$

$(x-1)(\boxed{2}x^2 - 3x + 5) = 0$

factor

$(x^2 - 3x + 10)$

$(x - \frac{2}{2})(x - \frac{5}{2})$

$\boxed{(x-1)(x-1)(2x-5)}$

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What are the rational roots of $3x^3 + 7x^2 + 6x - 8 = 0$?

Step 1: Find the factors of p & q

Step 2: Find possible roots $\frac{\pm p}{\pm q}$

Step 3: Test points to find one that equals zero

Step 4: plug into synthetic division

Step 5: Factor answer from synthetic division

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Step 3: Test Points

$1: 3(1)^3 + 7(1)^2 + 6(1) - 8$

$8 \neq 0$

$-1: 3(-1)^3 + 7(-1)^2 + 6(-1) - 8$

$-3 + 7 - 6 - 8$

$-10 \neq 0$

$2: 3(2)^3 + 7(2)^2 + 6(2) - 8$

$-2: 3(-2)^3 + 7(-2)^2 + 6(-2) - 8$

$-24 + 28 - 12 - 8$

$-16 \neq 0$

Mar 31-9:24 AM